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A NOVEL MATHEMATICAL MODEL OF AIRBORNE INFECTION

Nur Intan Syafinaz Ahmad[1]
Aishah Mahat[2]
Ahmad Khudzairi Khalid[3]
Nurhuda Ismail[4]
Muhammad Syawal Abd Halim[5]

[1]Faculty Science Computer and Mathematics, Universiti Teknologi MARA (UiTM), Malaysia, (E-mail: nurin395@johor.uitm.edu.my)
[2]Faculty Science Computer and Mathematics, Universiti Teknologi MARA (UiTM), Malaysia, (E-mail: aishahmahat@johor.uitm.edu.my)
[3]Faculty Science Computer and Mathematics, Universiti Teknologi MARA (UiTM), Malaysia, (E-mail: ahmad4829@johor.uitm.edu.my)
[4]Faculty Science Computer and Mathematics, Universiti Teknologi MARA (UiTM), Malaysia, (E-mail: nurhudaismail@johor.uitm.edu.my)
[5]Faculty Science Computer and Mathematics, Universiti Teknologi MARA (UiTM), Malaysia, (E-mail: syawal434@johor.uitm.edu.my)

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Abstract: *Every person can't see how infection through air will transfer from one person to other person. Those who're doing outdoor activities, they will be more exposed to many infections. Due to every infection can't be seen by eyes, most people will be ignoring the way how to prevent themselves from viruses via airborne transmission in public area especially using public transport. This paper focus to airborne transmission in train. This research used Wells-Riley Model because it is suitable for small population in a closed space. This model has been widely used for quantifying the infection risk assessment of respiratory infectious diseases indoor place. The risk of secondary infection is under control when the exposure time is less than 30 minutes for 150 passengers or less than one hour for 75 passengers and the reproduction number is estimated at 0.03. However, the reproduction number of second infections,* Q_A *is fairly insensitive to the number of passengers as the exposure time increase. But it is still under the skill of epidemic control as if* Q_A *is under 0.03. So, the spread of epidemic can be control. If* Q_A *more than 0.03 then procedures of prevention should be done.*

Keywords: *Disease, Wells-Riley Model, Airborne Transmission*

Introduction

Transmission of airborne is a transmission mechanism in which an infectious agent spreads as aerosols from reservoirs to humans and usually through the respiratory tract. Since this process cannot be seen with the naked eye, it has been discussed as the most interested subject for recent years as there is evidence that air quality in a closed space will have a major impact on the health of the person in the area. Although there are many infections that can be transmitted from one person to another through direct contact between infected and infectious individuals, but infection through air is something that can be treated with the exposed person who is in the infected person as it can be transmitted through such action cough or sneeze. There are many diseases that can be transmitted through airborne transmission such as flu, H1N1, Severe Acute Respiratory Syndrome (SARS), and Tuberculosis (TB). Traveling in public transport, usually in confined space will risk exposure to infectious pathogens. It appears that the risk of transmission of airborne infectious diseases in public transport such as buses, trains and taxis is higher than air transport according to Mohr (2012). Given that there are guidelines on contact detection after exposure to airborne pathogens while air travel exists, there is no guidance document found on contact detection in response to potential exposure to public transport, although European transport statistics show that the overall share of transport performance in 2007 on public transport almost twice as high (15.7%) as part of air transport performance (8.8%) according to Mohr (2012).

Moreover, it is decided to focus on train which is served by the Keretapi Tanah Melayu (KTMB). They have divided their services into two which is west coast line and east coast line. In this study the choice route is in east coast lines that are using KTM Intercity train. This is a service prepared for those who are live in Peninsular Malaysia and Singapore. The service is intended to carry people between stations in several cities, town and village that depend on the region. The train also powered by using diesel locomotives. This train has provided many facilities to make passenger who ride on it feel more comfortable. The facilities included a cabin with air conditioner, café, sleep coach and seat coach, and toilets. However, as in this project the seat coach is being chosen as in this coach usually will be filled by many passengers. Between two carriages it has a door to divide between carriage and each carriage has a window that can open and closed. As in train, with the crowded people in a carriage, would lead to the increasing of the chances to the number of infected persons by a factor of 2~3 and the peak of epidemic curve is up to 30 days earlier by assuming there are one infected person have ride on the crowded carriage according to Furuya (2007). Unfortunately, in public ground transportation, exposed passengers usually have one-time exposure as they do not receive information about the exposure other than by direct contact from either travel company or health authority since most passenger of ground transport vehicles do not use passenger name list. Besides, as for in a stationary place such as workplace or school, there are people potentially exposed to a patient spreading an infectious pathogen usually remain within the reach of one local health authority which can trace these contacts to initiate early diagnosis, make treatment and prevent for further spread.

When there are relate to airborne transmission, the Wells-Riley equation is the ideal model to estimate infectious cases to humans and its surroundings. Because of this model it allows the consideration of the small population impact on the train that can be attributed to a closed space to simulate the effects that are close to an infector. By understanding the potential of airborne transmission, it can provide important information to create a safe environment in public areas such as trains so that action can be taken to reduce the spread of airborne infections.

Problem Statement

Air quality in the building will have a great impact on the health of the people without them knowing. Most of them have been infected with airborne illnesses during their routine especially in public transport such as trains, buses and taxis because of the droplets of pathogens that expelled into the air due to coughing, sneezing and talking. The common infections that can spread by airborne transmission are anthrax (inhalational), chickenpox, influenza, measles, smallpox, cryptococcosis and tuberculosis. It is therefore important to calculate the potential risks posed by individuals infecting this public transport to the people around them

Objective

The core of this research is based on two objectives that are hoping to be as one of reference in solving problem arising from the airborne transmission in public ground transportation. The objectives are as follow:

a. To apply Wells-Riley model to estimate infection risk that are related to airborne transmission.
b. To calculate the secondary infection due to the airborne transmission in the train.

Scope and Significant

This project will only focus on calculating the potential risks of air transmission infections on KTMB trains located on inter-city railways on the east coast route. The results of this research can help specific bodies such as the Ministry of Health Malaysia (MOH) responsible for Malaysian health surveillance on air infections. The risk assessment of infection is very useful in understanding infectious disease infections. By predicting the risk of this disease to the public, it is important that it educate the public on health awareness especially in public areas so that the spread of airborne infections can be reduced. For the KTMB section, they can provide better coaches for their passenger's health. Table 1 below shows the definitions of terms and concept used in this study.

Table 1: Definition of Term and Concept

Term	**Definition**
Infector	Persons who are been infect and cause infection.
Susceptible person	Persons who are likely to suffer from a particular illness or be affected by a particular problem.
Airborne	Suspended in, transported by, or spread by air.
Infection	An agent or a contaminated substance responsible for one's becoming infected.
Ventilation rate	A space that contains the mechanical system or equipment used to circulate air or to replace stale air with fresh air.
Quantum generation rate	The number of infectious droplet nuclei required to infect the susceptible people in an enclosed space.

Literature Review

Wells-Riley model is used to estimate infection risk in a range of environments. The original studies Riley (1978) mentioned about the spread of measles in a Suburban elementary school where the epidemic was being investigated by a model that provides a basis for apportioning the chance of classmate to be infected while sharing same home room, or from exposure in school buses. According to Noakes (2006), she has studied the present's stochastic simulations using the Wells-Riley model to evaluate the infection risk and variability among small populations such as hospital patients. However, she has linked it with a simple multi-zonal ventilation model to show the airflow patterns and the approximate to an infected source on the effect of infection for an individual.

Further investigation by Liao (2005), he mentioned the respiratory infection getting the most pressing issues in the way to control them with current public health measure, the approaches like Well-Riley are used to prioritize the control measure effort. This model is to estimate the exposure concentrations in indoor environments whereas the inhalations of infection are reported. The researchers have set a simulation based on highly disseminated epidemic in Taiwan in an aircraft to meet the purpose of study which is to present an integrated-scale model that can help predict whether simple or some specific control measure are needed. There is also the study to estimate tuberculosis transmission risk in an internet café as in Japan, when people who are homeless frequently will choose internet cafe as a place to stay at night and this study are to quantify the public health risk of TB infection in such a facility using Well-Riley model based on the report of the TB outbreak in Internet Cafe by Furuya (2008).

Furthermore, some research has been also undertaken to determine the risk of airborne transmission in enclosed spaces, and the majority of previous studies root are from the work of Wells (1955) and Riley (1978), that are using the Wells-Riley model. Other than that, this has been applied to various risk analysis studies, including the evaluation of personal protective equipment Gammiatoni and Nucci (1997), tuberculosis risking buildings Nardell (1991) and the dispersion of Bacillus anthraces from envelopes where in this study they used the Wells-Riley model to estimate the exposure concentrations in postal facilities where cases of inhalation anthrax occurred and the risk for infection in various hypothetical scenarios of exposure to *B. anthracis* aerosolized from contaminated mail in residential settings Fennelly (2012) and also modelling the transmission of airborne infections in enclosed spaces Beggs (2003). However besides Well-Riley model, there are also researchers that used other method regarding to infections either the infections through air or direct contact. The other model for an epidemic model called SIR model that analyse model of the temporal behaviour of an infectious disease which is not extended in space.

Other researcher is Goldstein (2005) where they have made a model that was developed to calculate the age-specific risk of acquiring HBV infection, acute hepatitis B (illness and death), and progression to chronic HBV infection. HBV-related deaths among chronically infected persons were determined from HBV-related cirrhosis and hepatocellular carcinoma (HCC) mortality curves, adjusted for background mortality. The effect of hepatitis B vaccination was calculated from vaccine efficacy and vaccination series coverage, with and without administrations of the first dose of vaccine within 24 hours of birth (i.e. birth dose) to prevent prenatal HBV infection. However, the models made are quite complex as the outcomes would differ based on age when getting infection.

Methodology

Well-Riley Model

Wells-Riley model has been chosen in this paper to calculate the potential of airborne transmission of infection in train since it has been widely used as a tool to predict infection risk. There are several variables involve in the Wells-Riley model such as the number of infectors, breathing rate of susceptible people, the quantum generation rate (quanta/h) for the pathogen in infected person, the room ventilation rate, time of exposure, volume of ventilation space and the infectious unit produces per hour by infector. According to Riley, et al., (1978), Wells-Riley equation is stated in equation (1).

$$P = 1 - \exp\left\{ -\frac{Irqt}{M} \times \left[1 - \frac{v}{Mt} \times \left(1 - \exp\left(-\frac{Mt}{v} \right) \right) \right] \right\} \cong \frac{C}{S} \tag{1}$$

where

C	-	The number of disease cases
S	-	Number of susceptible people in train
I	-	I −Number of infectors
q	-	q −Infectious unit produces per hour by infector (quantum)
r	-	Breathing rate per person (m^3/h)
t	-	Time of exposure (h)
M	-	M −Room ventilation rate with fresh air (outdoor air supply rate)
v	-	Volume of ventilated space (m^3)

The term "quantum" is used to describe the "infectious dose" which defined as the number of infectious particles required to cause infection in $1-e^{-1}$ of a susceptible person when each susceptible individual breathes, on the average, one quantum of infectious particles. Duration of time exposure (h) was defined as the total number of hour the infectious person spent on the train, and number of susceptible person was taken as the number of passengers during one time in a train. The breathing rate per person was assumed to be 0.48 m^3/h. The volume of ventilated space is 60.3 m^3. Due to the steady-state exposure condition on this model from equation (1), several assumptions have been made:

a) Equal host susceptibility
b) Uniform sizes of droplet
c) Uniform ventilation
d) Homogeneous mixing of air
e) Elimination if infective particles being minimal compared with removal by ventilation

There are four steps in order to calculate the potential of airborne transmission of infection:

Step 1

The outdoor air supply rate, M were calculated using equation (2). However, since the fraction of indoor air that is exhaled by infector, f are still unknown, the outdoor air supply rate, M calculated from $\frac{M}{V}=13$ (Furuya, 2007).

$$M=\frac{nr}{f} \tag{2}$$

where

n	-	Number of people in ventilation space
f	-	Fraction of indoor air that is exhaled by infector

Step 2

The rate of quantum generation, q have been fixed as stated in equation (3) (Liao, 2005). This quantum generation rate is depended on biological characteristic and immunological state of the person who is susceptible.

$$q=\left[1-\left(\frac{1}{e^1}\right)(100)\right]=63.212\% \tag{3}$$

Step 3

Hence, substitute equation (2) and equation (3) into the equation (1) and we obtain

$$P=1-\exp\left\{-\frac{Iqft}{n}\times\left[1-\frac{vf}{nrt}\times\left(1-\exp\left(-\frac{nrt}{vf}\right)\right)\right]\right\}\cong\frac{C}{S} \tag{4}$$

Step 4

Finally, find the number of secondary infections, Q_A that happen after the susceptible were being together with the infector at the first place in a closed space. Assume that $I=1$ refer to an infector getting around susceptible person which the number of susceptible persons can be calculated $S=n-1$ (Furuya, 2007), then it can be expressed as stated in equation (5).

$$Q_A=(n-1)P \tag{5}$$

Implementation

The implementation of this study is from the secondary data which are from electronic article, journal and KTMB's officer. Table 2 shows the parameters that were used in this study.

Table 2: Secondary Data from Various Sources

Parameter	Base value
the number of susceptible people in train(S)	75(150)[(a)]
volume of ventilated space (m^3)	$60.3m^3$[(b)]
Total exposure time (t hours)	1 (0.1-2)
Breathing rate (r m^3/h)	0.48[(c)]
Fraction of indoor air exhaled by infected people (f)	0.016(d)
Number of infected people (I)	1

(a) From KTMB's officer

(b) Cited from (Donavan. M.L, 2004)
(c) Cited from (Liao. C.M, 2005)
(d) Determined using M/V = 13 (Furuya, 2007)

Data Implementation

This study used 150 respondents where 74 were sitting while the other 76 standing inside of the train. Equation (6) were used to calculate the fraction of indoor air exhaled by the infector, f, (Furuya, 2007).

$$\frac{M}{v} = 13 \tag{6}$$

where $v = 60.3m^3$, then $M = 13 \times 60.3 = 783.9m^3$.

After that, equation (2) and equation (7) were used to calculate the fraction of indoor air those are exhaled by infected people.

$$f = \frac{nr}{M} \tag{7}$$

Table 3: Fraction of Indoor Air Exhaled by Infected People in 1 Hour

Number of passengers, n	**Fraction of indoor air that are exhaled by infected people, f**
25	0.01531
50	0.03062
75	0.04592
100	0.061232
125	0.07654
150	0.09185

The result in Table 3 will be substituted in equation (4) as the total exposure is 1 hour which later produce the probability on infection for susceptible people as in Table 4.

Table 4: Probability of Infection for Susceptible People in 1 Hour

Number of passengers, n	**The probability of infection for susceptible people, P**
25	0.00038
50	0.00036
75	0.00036
100	0.00036
125	0.00036
150	0.00036

In order to get the reproduction number of second infections, Q_A were calculated using equation (5) which is the number of secondary infections that arise when having a single infectious case. It was introduced into susceptible people in a closed space by assuming there was 1 infector and the number of susceptible persons is $n-1$. As the average total time of exposure is 1 hour, the reproduction numbers for an infectious disease in the train with different number of passengers are as Table 5.

Table 5: The Reproduction Number of Second Infection Based On Number of Passenger for 1 Hour

Number of passengers, n	Reproduction number of second infections, Q_A
25	0.00912
50	0.01762
75	0.02664
100	0.03564
125	0.04464
150	0.05364

Since the train stops to drop and pick up passengers before it reaches the last destination, there will be a variation in duration of the exposure and the number of passengers. Therefore, the exposure was calculated within range 0.5 to 2 hours and the number of passengers were from 25 to 150 people as shown in Table 6.

Table 6: Reproduction Number of Second Infection Q_A

Time exposure, t	Number of passengers, n	Reproduction number of second infections, Q_A
0.5	25	0.00456
	50	0.00931
	75	0.01406
	100	0.01881
	125	0.02356
	150	0.02831
1.0	25	0.00912
	50	0.01765
	75	0.02664
	100	0.03564
	125	0.04464
	150	0.05364
1.5	25	0.0132
	50	0.0132
	75	0.0407
	100	0.0407
	125	0.0682
	150	0.08195
2.0	25	0.01776
	50	0.03626
	75	0.05476
	100	0.07326
	125	0.09176
	150	0.11026

Result and Discussion

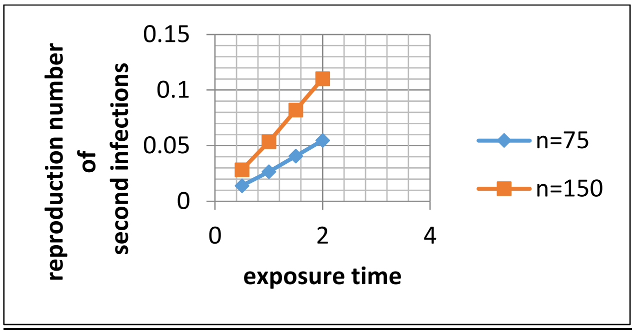

Figure 1: Graph Reproduction Number of Second Infection against Exposure Time

Figure 1 shows the difference between reproductions of second infections based on number of passengers with different exposure time. If the susceptible persons stay together with the infector, there will be an increase in the second infections linearly.

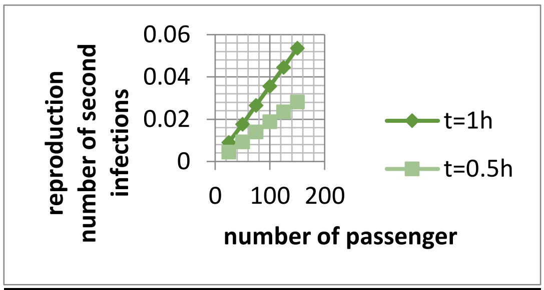

Figure 2: Graph of Reproduction Number of Second Infection against Number of Passenger

There is a relationship between number of passengers and infection as shown in figure 2. As the number of passengers increased from 75 to 150, the reproduction number of infections also increased from 0.02664 to 0.05364 for 1-hour commute. Meanwhile, for 0.5-hour commute, it increases from 0.01406 to 0.02381. As to estimate the influence of environmental parameters, the time of exposure and the number of passengers need to be changed. If the number of passengers is high, the infection from airborne transmission will spread rapidly compared to when the number of passengers is low.

This study found that the risk of secondary infection is under control since the reproduction number is 0.03 for the exposure time less than 30 minutes for 150 passengers and 1 hour for 75 passengers. However, the reproduction number of second infections Q_A is fairly insensitive to the number of passengers as the exposure time increase, it is still under the scale of epidemic control as if the reproduction number of second infections, Q_A under 0.03, the spread of epidemic can be control. Even though it has been demonstrated that it is hardly for a disease to growth in this environment, as long the Q_A is less than 0.03, provided this assumption holds true. However, if even a passenger in this space is not immune, there is always have the possibility that some individuals could become infected Giesecke (1994).

Conclusion and Recommendation

In conclusion, this study has indicated that the risk of having a secondary infection in the KTMB Train are very low. The reproduction number for the infection that have been calculated are 0.03 for exposure time less than 30 minutes for 150 passengers, which also means the number of secondary infections that happen when there is an infectious case around the susceptible people in the train are still under control. Therefore, there is a very light possibility for someone to get infected in the train. Since in this study the steady-state exposure conditions are applied in this model, the estimation of Q_A might vary from the real Q_A due to actual environment factors. However, the estimated result is still useful information for investigating the effects of the control of infection as there is insufficient data on the risk of infection transmission in a train compared to air transportation.

As recommendation for the future research, the other researcher can include other factor like wearing the mask and type of mask used and others. They also can calculate airborne infection in others public transportation.

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