USING FORD-FULKERSON ALGORITHM AND MAX FLOW-MIN CUT THEOREM TO MINIMIZE TRAFFIC CONGESTION IN KOTA KINABALU, SABAH

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Abstract: Traffic congestion is a major urban transportation problem which occurs when the traffic volume exceeds the capacity of existing road facilities. The occurring of traffic congestion is due to the freedom of owning private vehicle, poor traffic facility and unrestricted urban population growth. In this study, identification of maximum flow and bottleneck path in Kota Kinabalu, Sabah, Malaysia was carried out. The scope of this study is a network within the Central Business District of Kota Kinabalu. All the possible routes from source to sink will be established. In term of data collection, manual traffic count was used with the help of video recording. Ford & Fulkerson Algorithm will be applied to find out the maximum flow. Next, the max-flow and min-cut theorem will be used to determine the bottlenecks of a network. It allows the traffic engineer to decide which roadways facilities should be improved in order to minimize the traffic congestion problem.

Keywords: transportation, minimize traffic congestion, maximum flow, bottleneck, operation management.

Introduction

Traffic congestion is a major urban transportation problem which occurs when the number of traffic exceed the capacity of existing road facilities. Traffic Congestion can be an urban mobility problem which affects the economic productivity and the overall quality of life for many people. The occurring of traffic congestion is due to the freedom of owning private vehicle and unrestricted urban population growth. Besides, the limited land availability and difficulty of rebuild will cause slow improvement in traffic infrastructure (Teo et al., 2010).
Traffic congestion can be categorized into two types which are namely, recurring congestion and non-recurring congestion. Recurring congestion is known as peak hour traffic congestion which usually happens in the area of Central Business District (CBD); while, non-recurring congestion is caused by any number of unexpected events which will slow the traffic flow. The occurring of non-recurring congestion is due to accidents, road maintenance, road closure for events and natural disasters (McGoarty, 2010). Non-recurring congestion is a problem which is quite hard to handle because it is unpredictable. The happening of non-recurring congestion may cause the temporary closure of roadways, and thus reduce the roadway capacity. The decrement of road capacity will result in the decrease of maximal traffic flow, and will then slow the traffic vehicle’s speed dramatically. The application of the Ford-Fulkerson algorithm, followed by the max-flow and min-cut theorem will be used to identify the bottlenecks of the traffic congestion routes. These findings will also allow the traffic planar to decide which roadways facilities should be improved. Finally, the routes with low traffic congestion will be identified, and then classified as alternative routes for drivers. The improvement for the bottleneck path should put into practice which allows the traffic to move smoothly. Hence, the main objective of this study is to apply mathematical concepts, namely, Ford-Fulkerson algorithm and max-flow and min-cut theorem to find out the maximum flow and identify bottleneck path of the traffic congestion problems.

Problem statement and Study Scope

The problems of traffic congestion persist from day to day in Kota Kinabalu, Sabah (Murib Morpi, 2015). There are several reasons that have worsened the problems on traffic jams, viz., the increase of urban population, slow improvement of traffic facilities and behaviour of drivers on road. However, in this study the main criterion to be discussed is the performance of the facilities on road. Hence, the maximum flow and bottleneck paths were the part that were to be identified in order to minimize the problems of traffic congestion. Different paths or routes will have different maximum flow. The higher the maximum flow, the lower the degree of traffic jam. On the other hand, bottlenecks will limit the maximum flow of a route. Maximal-flow and minimum-cut theorem can maximize the flow in a network and take the maximum capacity of the route for optimization purposes on the traffic.

The scope of this study was a network of which the Masjid Bandaraya, Kota Kinabalu was set as source, and Kampung Air, Kota Kinabalu was set as the sink. All the possible routes from source to sink were established. The routes from Masjid Bandaraya to Kampung Air were selected because they are part of the Central Business District (CBD) for Kota Kinabalu. The demand of traffic was high, and the chances of occurring traffic congestion were higher than other locations. After the capacitated network was formed, the maximal flow would be computed by using the Ford-Fulkerson algorithm. Next, the max-flow and min-cut theorem would be used to determine the minimum cut value and the bottlenecks of the selected network.

Literature review

Traffic congestion in Kota Kinabalu

Teo et al. (2010) performed an optimization of traffic flow within an urban traffic light intersection with Genetic algorithm. Traffic light systems were built to control and ensure the smoothness of traffic flows at the intersections. The increase of traffic flow that could not be
solved by the traffic light system caused the long queues and congestions at intersection. The study was carried out in Kota Kinabalu, Sabah. The congestion problems in Kota Kinabalu became worsened due to the increase of on-road vehicles. New roads and lanes were suggested to solve the problem. However, the rebuilt of new traffic infrastructures became more difficult due to the limited land availability. Hence, the better solution was to study and design a traffic light controller to optimize the traffic flow. Data like queue length, green time, cycle time and amber time were observed and studied through simulations. Genetic algorithm was selected to find the optimized solution of traffic flow. By using Genetic algorithm, the current queue length was as the input and the optimized green time was the output. The simulation results showed that the Genetic algorithm was a good method to be used in traffic flow because it gave fast and good response to the change of queue length at the intersections.

**Bottleneck**

Yishui et al. (2015) performed a research of highway bottlenecks based on catastrophe theory. The analysis of Catastrophe theory involved the combination of VISSIM and Matlab. The VISSIM was for simulation and Matlab was for the data analysis. Based on Catastrophe theory, Yishui found out that there was a relationship between highway bottlenecks and congestions. The bottleneck may not cause congestion, but it did have great relationship with traffic flow. The influence of bottleneck might be less if the accumulation of traffic flows happened before the inflection point. In contrast, the influence of bottleneck to the traffic flow would suddenly increase if the accumulation of traffic flow happened after the inflection point. The study of catastrophe theory could be very helpful for traffic control and phenomena prediction.

However, de Souza et al. (2015) published a paper with title: “A solution using cooperative routing to prevent congestion and improve traffic condition”. They stated that the sudden traffic congestion during peak hours was due to the increase of vehicles in the areas with bottlenecks. The current solution was to re-routing vehicles to avoid the congested area, but it did not solve the problem in the long term. This was because re-routing vehicles with alternative routes would create new bottlenecks. Hence, they proposed an intelligent traffic cooperative routing application called SCORPIAN to improve overall routing utilization and reduced travel cost by avoiding vehicles getting stuck in a congestion.

**Capacity estimation**

Suresh & Umadevi (2014) stated the fundamental details of traffic flow and several methods of estimation of capacity for Indian urban road. The major types of estimation were be classified under two broad categories as Direct Empirical Methods and Indirect Empirical (Simulation) Methods. Due to the complexity and high volume of traffic on Indian urban roads, it was appropriate to model the flow of parameters and adapt direct empirical methods for estimation of capacity. Direct empirical methods used the observed fundamental traffic characteristics like headway, volume and speed for capacity estimation. Three methods, which were the Headway method, Observed volume method and Fundamental diagram method were employed and compared. The Headway method was found to have achieved the highest capacity estimation.
Maximum flow algorithm

Moore et al. (2013) studied the maximum flow in road networks with speed dependent capacities application to traffic in Bangkok. A traffic flow problem took edge weights which represented road capacities (maximum vehicles per hour) that were functions of the traffic speed (kilometer per hour) and traffic density (vehicles per kilometer). To estimate road capacities for a given speed, empirical data on safe vehicle separations for a given speed were used. A modified version of the Ford-Fulkerson algorithm was developed to solve maximum flow problems with speed-dependent capacities, with multiple source and target nodes. It was found that the maximum safe traffic flow occurred at a speed of 30 km/hr.

Gebreanenya (2016) studied the maximum flow problem of Ethiopian Airlines. In his study, the maximum flow problem and its solution algorithm, Ford and Fulkerson algorithm was discussed. The objective in the study was to reschedule the maximum number of passengers from cancelled flights from Addis Ababa to New York via the other flights. By using the Ford-Fulkerson algorithm, at least one arc in the augmenting path was saturated. The Ford-Fulkerson algorithm solution showed that different augmenting paths and different number of augmenting paths might exist, eventhough they had the same maximum flow value.

However, Takahashi (2016) studied the simplest and smallest network on which the Fold-Fulkerson Maximum Flow procedure might fail to terminate. Ford-Fulkerson’s labeling method always terminated for networks that had an edge with rational capacity. However, it might fail to terminate in the sense that it had an infinite sequence of flow augmentations. The results suggested that Ford-Fulkerson algorithm might fail to terminate networks with real-valued capacities since the network with edges contained subgraph homeomorphic and irrational capacities.

Besides, Ford- Fulkerson algorithm can be applied in other fields like liquid flow through pipe, information in communications network, current flow in electrical circuit, production factory with various tools and others (Mützell & Josefsson, 2015). Neto & Callou (2015) proposed an approach based on Ford-Fulkerson algorithm to maximize the bandwidth usage of computer networks. A flow optimization application based on Ford-Fulkerson algorithm was proposed which allowed mitigating congestion problem and increasing bandwidth usage. Different analysis for different network scenarios were presented to show the applicability of the proposed approach. The first scenario was to analyze and maximize bandwidth usage which was composed of switches and host connected via Ethernet. The second scenario was considering a Fiber-optic communication network. Throughout the experiment conducted, the results showed that the better improvements were identified once a big amount of traffic was applied.

Wang & Wang (2014) performed a study on Highway Traffic Capacity based on the maximum flow algorithm. Firstly, they focussed on the basic theory of highway traffic capacity, followed by the method of maximum flow algorithm to calculate the highway capacity. The road capacity referred to the maximum number of vehicles at a particular section of the road on traffic route map analysis. They then formed the traffic capacity of multi point and multi destination network. It included the starting point and finish point which were the outset and destination. The vertices were the road intersection and the edges in the network were the road networks. Then, these were transformed to single starting point and single end point network problem. Wang & Wang (2014) then performed network
simplification process to obtain the maximum flow of the network. They concluded that the results were the same as the results of the labeling method. Contribution of this paper was the transformation of the highway network capacity into a specific mathematical model, and then solved by a simple maximum flow algorithm.

Zhao & Meng (2012) presented an improved algorithm for solving maximum flow problem. The existing algorithms involved lots of steps and complicated calculations for solving maximum flow problems due to the improper selection of augmenting path. Hence, the proposed improved algorithm was the use of a new concept that involved dividing the area and the degree of vertex. As a result, the proposed concept of solving maximum flow problem did not involve the labeling method, and the entire process only needed to draw a diagram. According to Roughgarden (2016), the optimal maximum flow could be achieved with different number of augmenting paths. The obtained maximum flow by Ford-Fulkerson Algorithm had remained in an integer form since all the parameters in the network graph are in integral, so the maximum flow stays integral.

**Max-flow Min-cut Theorem**

Baruah & Baruah (2013) presented applied minimum cut maximum flow using cut set of a weighted graph to the traffic flow. A weighted graph was a resulting graph with a real number which served as a structural model in transportation. The traffic control strategy of minimal cut and maximum flow was to minimize the number of edges in a network and maximum capacity of vehicles which moved through these edges. With a minimal cut in the traffic network, it allowed minimum waiting time of traffic participants for a smooth and uncongested traffic flow.

Petrov et al. (2017) presented an effective algorithm for finding a minimum cut of any transport network. The main objective was to localize the pipeline bottleneck of a transport network. Petrov described an algorithm for the solution of generalized problem which inflows and outflows are assuming constant in an arbitrary network nodes. The algorithm was used to find the minimum cut and maximum flow of the network. Dual problem involved to find bottlenecks and the flow value could be less than or equal to maximum flow. The finding was the sum of capacity constraints of edges for such a cut could be larger than the minimum of the dual problem. The proposed algorithm could effectively find out the bottlenecks in constant inflows and outflows condition.

Dong & Zhang (2011) presented a research on method of traffic network bottleneck identification based on Max-flow Min-cut Theorem. It allowed the weak section of the road to be identified and provided a solution for the traffic problem. The traffic networks must be formed into the map of graph theory before identification of bottleneck. They applied the Max-flow Min-cut Theorem to find out the bottleneck of the network. This theorem stated that the minimum cut was the smallest capacity of the road section. Therefore, it determined the maximum capacity of the whole network. The identified weak parts of the road allowed traffic planar to know that which parts of the road needed to be widened. They further proposed a way to solve traffic bottleneck which was to increase road lines. Simulation software, ExSpect was applied to perform simulation on the new network with which road line was added to test the efficiency of the solution. The results showed that identification of bottleneck based on Max-flow Min-cut Theorem could thus find out the bottleneck effectively.
Methodology

Phase 1
Collection of data

There were two different methods to conduct the traffic volume counts which are the manual counts and the automatic counts (Hamsa, 2013). The most common method of collecting traffic volume data would be the manual method of traffic volume count, which involved a group of people recording the number of vehicles passing on a pre-determined location, using tally marks in inventories (Bharadwaj et al., 2016). There are also several methods to estimate the roadway capacity. These methods can be categorized into direct empirical method and indirect empirical method (Suresh & Umadevi, 2014). The direct empirical method made use of the observed data to estimate the capacity directly. However, in the case of indirect empirical method, it would use the observed data and some computer softwares to run simulations which involved complex simulation models. However, the estimated capacity by the simulation model might not be accurate.

Phase 2
Formation of network graph

Before maximum flow algorithm is being applied in this study, a network graph will be needed. To form a network graph, there are several data needed such as the name, direction and capacity of the routes within selected scopes. The intersections are appointed as the nodes of the network graph, the path that connected between the intersections are the edges with direction and the capacity of the road sections assigned according to the direction of the edges. With all the nodes, edges, directions and capacities, a directed network graph can be formed. Network is formed by edges and connected to the nodes. A directed capacitated network graph was formed with the capacity that obtained through observation data. It used to compute maximal flow. First, a directed capacitated network graph was formulated with all the edges and nodes. Each of the edges has a non-negative capacity, $c(u,v) \geq 0$ and flows, $f(u,v)$ that cannot be more than capacity of the edge. Single Source and Single Sink is discussed in this study. Hence, the network graph will only have one source node, $s$ and one sink node, $t$ which are the starting node and ending point. A directed capacitated network must fulfill the conditions below: First, the capacity constraint, $\forall (u,v) \in E f(u,v) \leq c(u,v)$ which flow of the edges must not exceed its own capacity. Then, the next condition is skew symmetry, $\forall u,v \in V, f(u,v) = -f(v,u)$ which net flow from $u$ to $v$ and from $v$ to $u$ must be opposite to each other. Lastly, flow conservation constraints, $\forall u \in V: u \neq s \text{ and } u \neq t \Rightarrow \sum_{(s,u)} f(s,u) = \sum_{(v,t)} f(v,t)$ is the net flow to a node is zero except source node and sink node and the flow from the source node must be equal to the flow at the sink node.

Phase 3
Maximum Flow Algorithm: Ford Fulkerson Algorithm

Maximal flow in a capacitated flow network is total flow from a source node to a sink node. First, find an augmenting path from source node to sink node where each edge
Phase 4

Max-Flow & Min-Cut Theorem

Ford-Fulkerson algorithm was a published algorithm in 1956. In 1970, Yefim Dinitz implemented Edmonds-Karp algorithm which is an improved algorithm compared to Ford-Fulkerson algorithm. Edmonds-Karp algorithm makes use of breath-first search to find augmenting paths. Next, the Dinic’s blocking flow algorithm is implemented and it builds layered graph with breadth-first search on the residual graph before finding augmenting paths. In 1986, push relabel algorithm is published by using the concept of preflow. It uses local operation which allows the method to be faster in practice (Goldberg & Tarjan, 2014). The Maximum-Flow and Minimum-Cut Theorem is used to find the maximum flow of the directed capacitated network (Dwivedi & Yu, 2013). The maximum amount of flow that passes through from source node to sink node is equal to the total weight of the edges in minimum cut. \[ \text{Max} \{\text{val}(f) ; f \text{ is a flow}\} = \text{min}\{\text{cap}(S) ; s \text{ is an (s,t) - cut}\} \] (Goldberg & Tarjan, 2014).

Research Design

The framework of the research design in Figure 1 showed the flow on minimizing the traffic congestion problems. There were two types of traffic congestions occurred, namely, the recurring and non-recurring, whereby only the recurring congestion was discussed in this paper. Cost, capacity, time and distances were the typical parameters. However, the capacity was the main parameter used in this study. The selected network was a directed capacitated network with a single source and a single sink. Based on the network, the Ford-Fulkerson algorithm viz. the Maximum Flow and Minimum Cut Theorem was selected to find the maximum flow and bottleneck paths.
Results and Discussion

Data collection

In this study, the needed data was the volume of vehicles passing through the respective selected routes. Due to costs incurred and budget constraints, the manual traffic count was selected as the most suitable method compared to the automatic count. Therefore, the traffic volume count should be performed at the selected intersections or roadways. The tools that were needed for manual traffic counts were tally sheets, stopwatch, pen and paper.

The traffic data collection would involve different time intervals like hourly, daily, or monthly. The data in this study was collected hourly so that it matches with the roadway capacity estimation method. Since the data was collected in an hourly mode, a particular time interval was chosen to collect the traffic data of a higher accuracy. The chosen time intervals were the peak hours in the mornings of weekdays, instead of weekends. The peak hours in the mornings around 7 am to 8 am were the most suitable time because of the high traffic volume. Suppose that the data was initially collected on a spot with tally sheets and pen, but it was then later changed to video shooting due to the problem of data accuracy. The data was then collected through video clips by using a camera with a tripod stand. The camera was mounted on the tripod stand, and it would be located at a higher place so as to have a wider and clearer view. The video clips with a replay function would allow the collected data to be reviewed again repeatedly.
The direct empirical method was selected in this study due to less cost, easier to perform and would expect better accurate results of the estimated capacity. There are three approaches in direct empirical methods which are headways approach, volume approach and fundamental diagram approach. Among these approaches, the volume approach was chosen as the main approach to estimate capacity of the selected roadways. The data needed for estimation were roadway width, traffic volume, headway, speed and density.

Table 1 below displayed the Passenger Car Unit (PCU) values for different categories of vehicles by referring to the Urban Traffic System (Hamsa, 2013). The estimation of capacity by the volume approach made use of the Passenger Car Unit (PCU) value as in Table 1.

<table>
<thead>
<tr>
<th>Class of vehicles</th>
<th>Urban Road PCU value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car, Taxi, Light Goods Vehicle (LGV)</td>
<td>1.00</td>
</tr>
<tr>
<td>Motorcycle, Scooter</td>
<td>0.75</td>
</tr>
<tr>
<td>Medium Goods Vehicle (MGV) or Heavy Goods Vehicle (HGV)</td>
<td>2.00</td>
</tr>
<tr>
<td>Bus, Truck</td>
<td>3.00</td>
</tr>
</tbody>
</table>

Table 2 below showed the composition of traffic based on the different class of vehicles passing through the related network. Hence, the total capacity of the selected locations in PCU can thus be calculated. In Table 2, different nodes had been assigned to the selected paths from the source node to the sink node. Each of the selected paths with the composition of different category of vehicles was displayed. Vehicle counts were classified as car, motorcycles, Large Goods Vehicle (LGV), Medium Goods Vehicle (MGV) or Heavy Goods vehicle (HGV), and buses or trucks. The composition of traffic in Table 2 was the maximum flow in 5 minutes interval of the peak hour in morning.

<table>
<thead>
<tr>
<th>No</th>
<th>Location name</th>
<th>From</th>
<th>To</th>
<th>Car</th>
<th>Motorcycle</th>
<th>MVG &amp; HGV</th>
<th>Bus &amp; Truck</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Jalan Tun Fuad Stephen 1</td>
<td>s</td>
<td>V1</td>
<td>171</td>
<td>39</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>Jalan Pasir</td>
<td>s</td>
<td>V2</td>
<td>114</td>
<td>18</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>Jalan Tun Fuad Stephen 2</td>
<td>V1</td>
<td>V5</td>
<td>173</td>
<td>32</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Jalan Tuaran 1</td>
<td>V2</td>
<td>V3</td>
<td>162</td>
<td>28</td>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>Jalan Kompleks Sukan &amp; Jalan Bunga Nasar</td>
<td>V3</td>
<td>V4</td>
<td>114</td>
<td>5</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>Jalan Kompleks Sukan &amp; Jalan Bunga Nasar</td>
<td>V4</td>
<td>V3</td>
<td>56</td>
<td>11</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>Jalan Istiadat</td>
<td>V4</td>
<td>V1</td>
<td>58</td>
<td>15</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>Jalan Istiadat</td>
<td>V1</td>
<td>V4</td>
<td>50</td>
<td>8</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>Jalan Tuaran 2</td>
<td>V3</td>
<td>V6</td>
<td>130</td>
<td>19</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>Jalan K.K Bypass</td>
<td>V5</td>
<td>V7</td>
<td>90</td>
<td>18</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>Jalan Tuaran 3</td>
<td>V6</td>
<td>V9</td>
<td>169</td>
<td>13</td>
<td>19</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>Jalan Tunku Abdul Rahman 1</td>
<td>V7</td>
<td>V8</td>
<td>57</td>
<td>20</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>13</td>
<td>Jalan Tunku Abdul Rahman 2</td>
<td>V8</td>
<td>V9</td>
<td>62</td>
<td>12</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>14</td>
<td>Jalan Kemajuan</td>
<td>V9</td>
<td>V10</td>
<td>153</td>
<td>32</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>15</td>
<td>Jalan Laiman Diki</td>
<td>V7</td>
<td>t</td>
<td>91</td>
<td>21</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>16</td>
<td>Jalan Coastal</td>
<td>V10</td>
<td>t</td>
<td>81</td>
<td>9</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>
In the volume approach, the selected maxima model will be employed to estimate capacity. The flows in each 5 minute time intervals were extracted from the peak hours in the morning. A 5 minute interval with maximum flow was identified. The peak flow rate was then multiplied by 12 in order to get the estimated capacity per hour. Similar steps were done for the other locations, and displayed in Table 3. The maximum capacity obtained by this method was 2913 PCU (highlight in yellow), and the minimum capacity obtained was 843 PCU (highlight in blue).

Table 3: Capacity estimation using Selected Maxima Model of the selected locations (5 min time slice)

<table>
<thead>
<tr>
<th>Location</th>
<th>No</th>
<th>Max Flow in 5 Minutes in PCU</th>
<th>Total Capacity in PCU</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>226.25</td>
<td>2715</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>175.5</td>
<td>2106</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>220</td>
<td>2640</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>242.75</td>
<td><strong>2913 max</strong></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>130.75</td>
<td>1569</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>70.25</td>
<td><strong>843 min</strong></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>82.25</td>
<td>987</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>74</td>
<td>888</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>156.25</td>
<td>1875</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>118.5</td>
<td>1422</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>231.75</td>
<td>2781</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>87</td>
<td>1044</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>95</td>
<td>1140</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>211</td>
<td>2532</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>120.75</td>
<td>1449</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>95.75</td>
<td>1149</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2: Network graph with estimated capacity
Figure 2 showed a network graph from source node (Masjid Bandaraya, Kota Kinabalu) to sink node (Kampung Air, Kota Kinabalu) with the estimated capacity from Table 3. Figure 2 represented a 12-node and 16-edge network with one source and one sink.

**First type of augmenting path by Ford-Fulkerson Algorithm**

![Figure 3: First Augmenting Path (First type)](image)

The first augmenting path of the first type was from $s \rightarrow V_1 \rightarrow V_5 \rightarrow V_7 \rightarrow t$ (upper red line in Figure 3) which carried a maximum flow of 1449 vehicles per hour.

![Figure 4: Second Augmenting Path (First type)](image)

The second augmenting path of the first type was from $s \rightarrow V_2 \rightarrow V_3 \rightarrow V_6 \rightarrow V_9 \rightarrow V_{10} \rightarrow t$ (lower red line in Figure 4) and carried a maximal flow of 1149 vehicles per hour.
Figure 3 and 4 were the steps of augmenting paths using the Ford-Fulkerson algorithm. From Figure 5, the total maximum flow value would be the sum of these two augmenting paths, and hence, amounting to 2598 vehicles per hour.

*Second type of augmenting path by Ford-Fulkerson Algorithm*

The first augmenting path of the second type was from \((s \rightarrow V_1 \rightarrow V_5 \rightarrow V_3 \rightarrow t)\) (upper red line) which carried a maximum flow of 1449 vehicles per hour.
The second augmenting route of the second type carried 678 vehicles per hour from \((s \rightarrow V_2 \rightarrow V_3 \rightarrow V_6 \rightarrow V_9 \rightarrow V_{10} \rightarrow t)\) (red line).

The third augmenting path from \((s \rightarrow V_1 \rightarrow V_5 \rightarrow V_7 \rightarrow V_8 \rightarrow V_9)\) (lower red line) carried a maximal flow of 471 vehicles per hour.
The maximum flow of the network graph in Figure 5 was 2598 of vehicles per hour. From Figure 9, the total maximum flow value would be the sum of the three augmenting paths of the second type, and hence, amounting also to 2598 vehicles per hour. From Figures 5 and 9 respectively on the augmenting paths, it could be seen that different number of augmenting paths and different augmenting paths had been obtained, however the value of maximum flow of the network was the same. This implied that without referring to which augmenting path was better, the different number of augmenting paths would all give to the same value of maximum flow.

**Max-Flow & Min-Cut Theorem**

The cut capacity and the sum of the capacities of its arcs were the same. Bottleneck capacity which was the minimum residual capacity of any edge in the network. The cut with the
smallest capacity gives the maximum flow in the capacitated network. From the Figure 4, the blue line stood for the minimum cut with the smallest capacity which was the same as the maximum flow that was computed by the Ford-Fulkerson algorithm. The bottleneck routes of the network in Figure 4 were Jalan Laiman Diki and Jalan Coastal. Traffic planar should take the bottleneck paths as one of the consideration to minimize the traffic congestion in Kota Kinabalu.

Discussion

The Ford-Fulkerson algorithm can exactly terminates a network graph with a feasible flow. It will continue to progress further the augmenting path if it has more eligible augmentation paths. It will halt if there is no eligible augmentation path exists. Even though from the history of maximum flow algorithm, the Ford-Fulkerson is considered as one of the simplest, yet a friendly user’s algorithms. Through this study on transportation problems like traffic congestions, without referring to which augmenting path was better, different number of augmenting paths and different augmenting paths obtained would all give to the same value of maximum flow. This finding is similar to Gebreanenya (2016) where he had published a paper that had applied Ford-Fulkerson algorithm in solving the maximum flow problem of Airlines.

Even though there are still have some current published papers that are still using the Ford-Fulkerson algorithm to find outputs, the Ford-Fulkerson algorithm can be used to compute an optimal solution, but it is still considered as an old algorithm, giving slow outputs when compared to other algorithms. Moreover, Max-Flow and Min-Cut Theorem is an s-t cut of a network graph. It can be used to identify the bottlenecks and maximum flow of the network graph. The cut is performed by cutting down the path between s-t vertices. Different cuts have different capacities. The cut with the minimum cut capacity is the maximum flow of the network. It is a direct and simple method to get the maximum flow. A further study of this algorithm can be done so that the solution can be obtained with less iterations and in a shorter time.

Conclusion

The maximum flow for the capacitated network with 12 nodes and 16 edges of the selected scope in this study was 2598 vehicles per hour. The traffic planar who designed road and transportation networks should take concern on the bottleneck paths like Jalan Coastal and Jalan Laiman Diki. However, the traffic volume might change from time to time depending on the day of the week and the period of activity. Therefore, the capacity in this study will not be constant at all times. Further research is thus suggested especially on the part of capacity estimation whereby some data like vehicle speed, width of the road, number of lanes, and others that can be included in for capacity estimation.

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